MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

EME4086 – FINITE ELEMENT METHOD

(ME)

21 OCTOBER 2019
2.30 p.m. - 4.30 p.m.
(2 Hours, Open Book)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 5 pages with 4 Questions only.
2. Attempt **ALL FOUR** questions of 25 marks each.
3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows a schematic sketch of a lamp post which supports six lamps at the top. Weight of each lamp is 150 N. The pole is made of a hollow tapered cylinder with constant thickness of 0.01 m. Outer diameter of the pole at point A is 0.4 m and outer diameter at point B is 0.2 m. Young's modulus of the post is 120 GPa. Assuming that total weight of lamps as a point load at the top and ignoring weight of the pole itself, do the following:

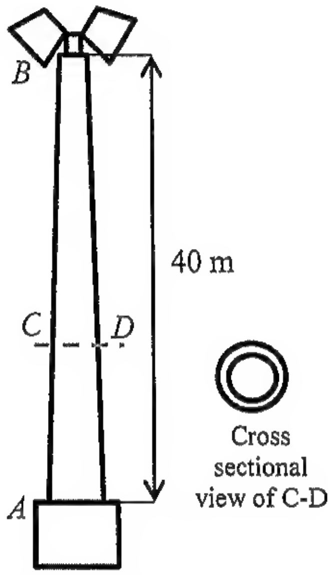

Figure Q1: Simplified sketch and dimensions of a lamp post.

a. Model the lamp post by using 2 equally spaced 1- dimensional finite elements. Show the element numbers, nodes, simplified dimensions and the boundary conditions in the model. **[3 marks]**

b. Write down the global finite element equation for the model in the form of $[K][U] = [F]$. Substitute numerical values into the global finite element equation, where applicable. **[8 marks]**

c. Determine changes in the global $[F]$ matrix, if the temperature increases from 21°C at coldest night time to 39°C at hottest afternoon time. Given coefficient of thermal expansion, $\alpha = 12 \times 10^{-6}/^{\circ}C$. **[5 marks]**

d. Based on the new $[F]$ matrix calculated in part c. above, write down the condensed global finite element equation. Then, determine the deflections and reaction forces at the unknown nodes for the model. **[9 marks]**

Continued ...

Question 2

One-dimensional problem involving heat conduction with heat generation can be expressed by the following differential equation:

$$-k\frac{d^2T}{dx^2} + Q = 0 \qquad for \quad 0 < x < 1$$

where k is the thermal conductivity, $T(x)$ is the temperature, and Q is heat generated per unit length. Q, the heat generated per unit length, is assumed to be constant. Two essential boundary conditions are given at both ends: $T(0) = T(1) = 0$. Do the following:

a. Show, in detailed steps, that the weak form for the nonlinear differential equation above is given as:

$$B(v,u) = \int_0^1 \left(\frac{dT}{dx}\right)\left(\frac{dv}{dx}\right) dx$$

$$l(v) = \frac{Q}{k}\int_0^1 v\, dx$$

[8 marks]

b. Find a one-parameter approximate solution using Ritz method.

[8 marks]

c. Find a one-parameter approximate solution using Galerkin method

[7 marks]

d. Compare the results obtained in parts b. and c. with the exact solution given by:

$$T(x) = \frac{Q}{2k}x(1-x)$$

and comment on the selection of the trial function for the problem. **[2 marks]**

Hint: Choose ***only one*** valid trial function from the following to answer:

$$\emptyset_i = x^i \qquad \emptyset_i = x^i(1-x) \qquad \emptyset_i = (1-x)^i$$

Continued ...

Question 3

A truss structure supports a vertical weight $F = 40$ kN as shown in **Figure Q3**. The structure is wall mounted at nodes 1 and 2 while node 4 is supported by a roller. Cross sectional area of each member is 0.005 m^2, $L = 15$ m and the Young's modulus is 100 GPa. Do the following:

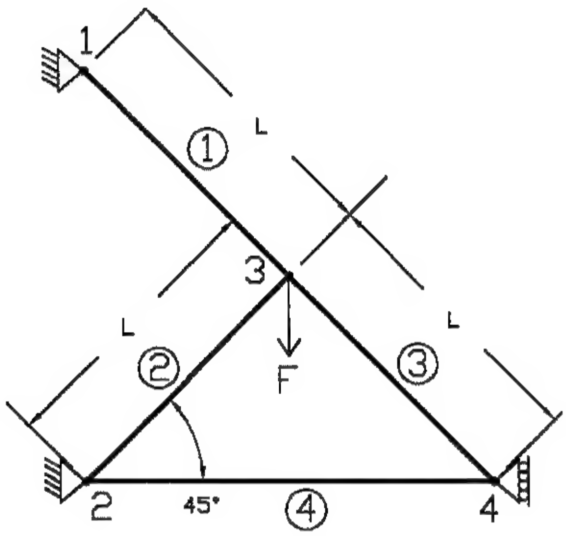

Figure Q3: Truss structure supporting a weight, F.

a. Determine the unknown nodal displacements. **[17 marks]**

b. Determine internal force for each element of the structure and clearly indicate whether it is under tension or compression. **[8 marks]**

Continued ...

Question 4

Figure Q4 shows a 1 m^2 square plate that is discretized by using two constant strain triangular (CST) elements. Global and element numbers are shown in the **Figure Q4.** Global nodes 2, 3 and 4 are supported by rollers while global node 1 is fixed to the ground. Nodes 2 and 3 are subjected to 50 kN of load, individually. Young's modulus and Poisson's ratio of the plate are 70 GPa and 0.3, respectively. Thickness of the plate is 0.1 m. Assume plane stress condition and do the following:

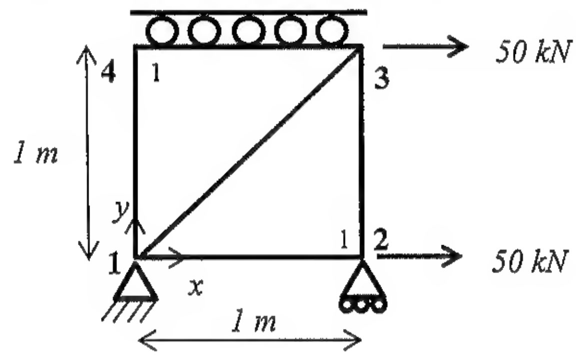

Figure Q4: A square domain subjected to load.

a. Write down the elements' stiffness matrices in the form of $k = V\ [D^T]\ [C]\ [D]$. Substitute numerical values into the matrices. Take node 1 as the origin.

[10 marks]

b. Write down the condensed finite element equation in the form of $[K][U] = [F]$. Stiffness matrix for the elements is:

$$\begin{bmatrix} 5.2 & -1.3 & -3.8 & -2.5 & 1.2 & 1.3 \\ -1.3 & 1.3 & 0 & 1.3 & 0 & -1.3 \\ -3.8 & 0 & 3.8 & 1.2 & -1.2 & 0 \\ -2.5 & 1.3 & 1.2 & 5.2 & -3.8 & -1.3 \\ 1.2 & 0 & -1.2 & -3.8 & 3.8 & 0 \\ 1.3 & -1.3 & 0 & -1.3 & 0 & 1.3 \end{bmatrix} \times 10^9$$

[6 marks]

c. Solve for the unknown nodal displacements. **[6 marks]**

d. Estimate displacement of midpoint of edge 1-2. **[3 marks]**

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